Optimal Demand Response

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Outline

Caltech smart grid research

Optimal demand response

SOUTHERN CALIFORNIA EDISON NSF arpa·e CISCO SYSTEMS THE OKAWA FOUNDATION

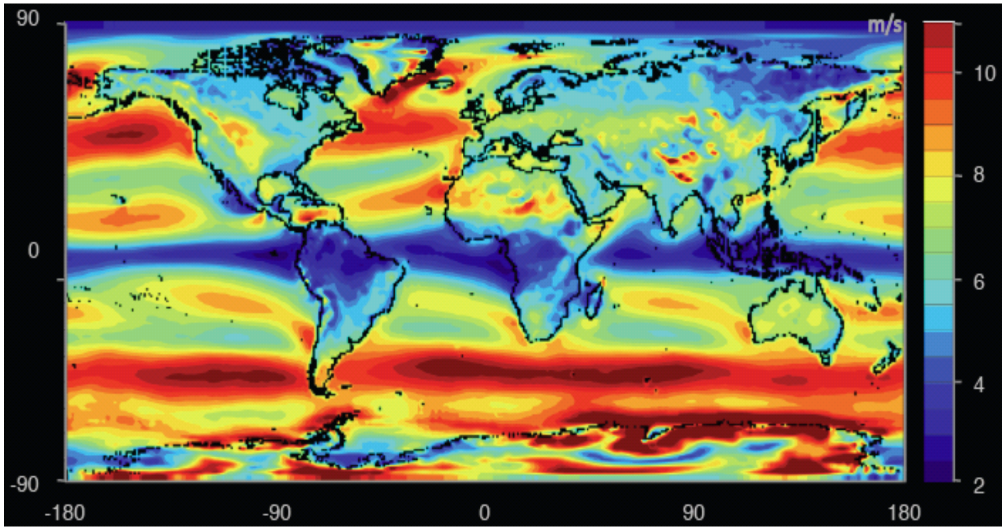
Global trends

1 Exploding renewables

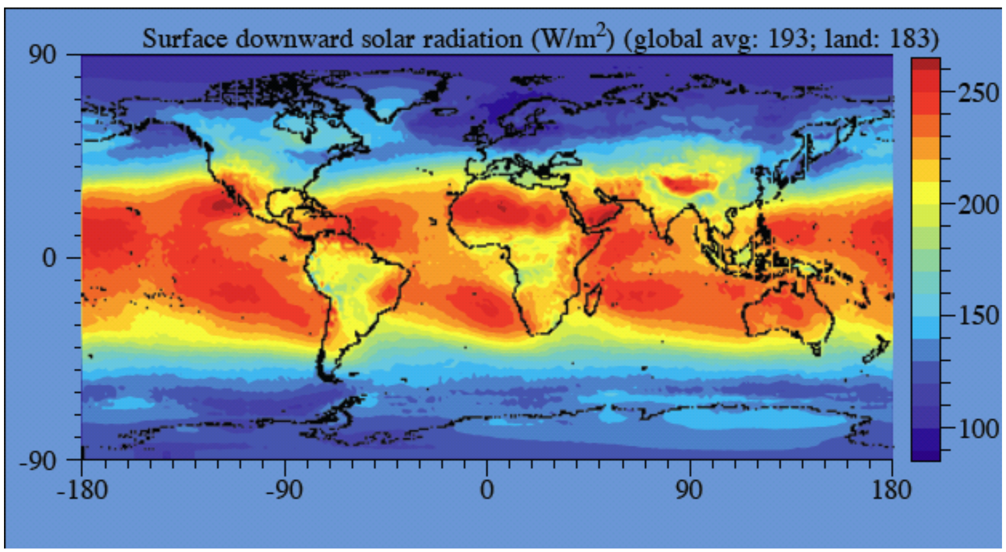
- Driven by sustainability
- Enabled by policy and investment

2 Migration to distributed arch

- 2-3x generation efficiency
- Relief demand on grid capacity



Wind power over land (exc. Antartica)
70 – 170 TW



Solar power over land
340 TW

<u>Worldwide</u>

energy demand:
16 TW

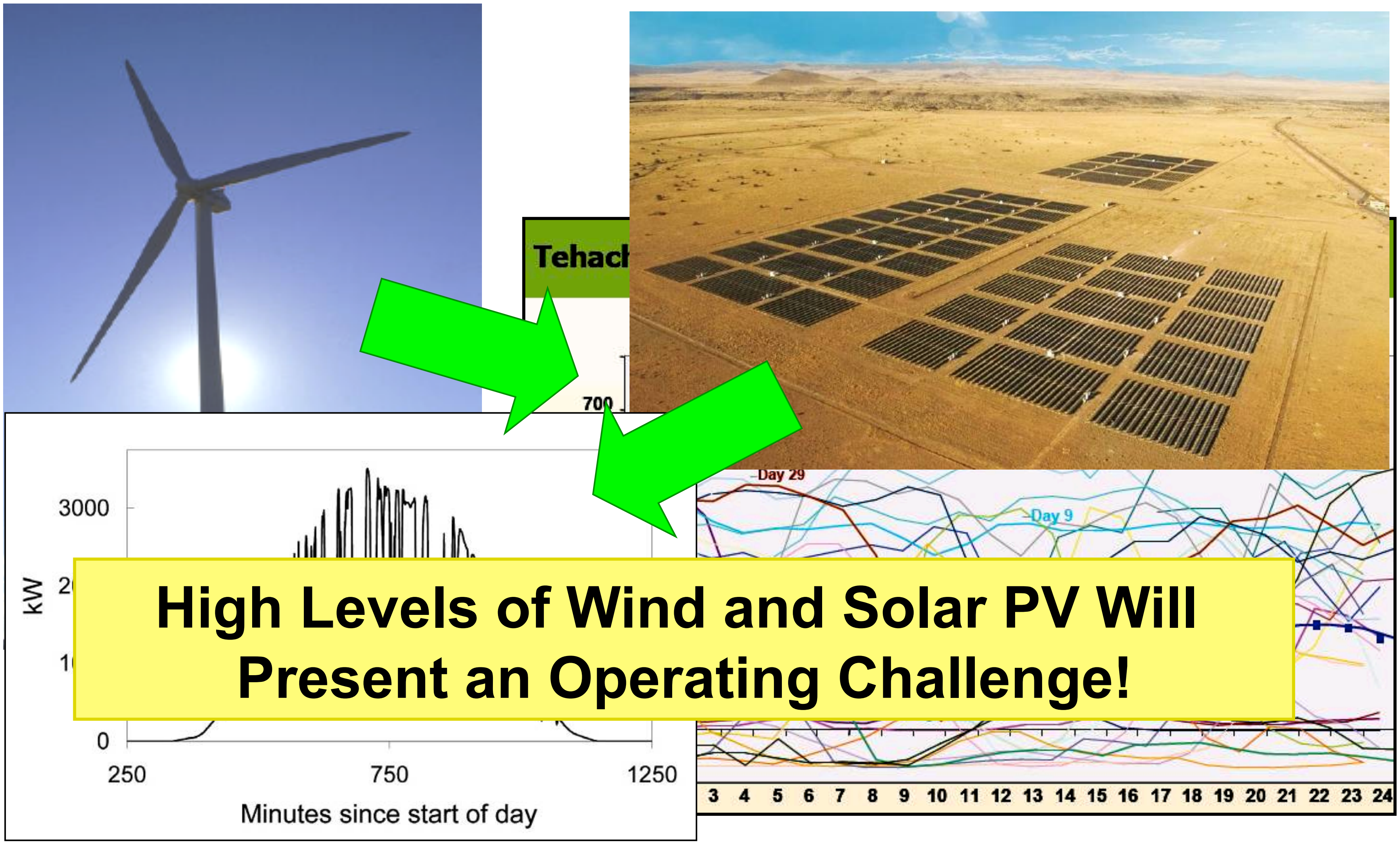
electricity demand:
2.2 TW

wind capacity (2009):
159 GW

grid-tied PV capacity (2009):
21 GW

Source: Renewable Energy
Global Status Report, 2010
Source: M. Jacobson, 2011

Key challenge: uncertainty mgt



Source: Rosa Yang, EPRI

Large-scale active network of DER

	#nodes	capacity per node	total capacity	completion time	remarks
SCE	500	1 MW	500 MW	2015	SCE Commercial Rooftop Solar
CA	175,000	10 kW	1.75 GW	2016	CA Solar Initiative
SCE	400,000	2 kW	800 MW	--	10% penetration of SCE residential customers
CA	1,000,000	3 kW	3 GW	2017	CA Million Solar Roofs Initiative
CA	--	--	25 GW	2020	CA Renewable Portfolio Standard
US	--	--	3 TW	2035	Obama's goal for clean energy

DER: PVs, wind turbines, batteries, EVs, DR loads

Large-scale active network of DER

	#nodes	cap			remarks
SCE					Rooftop
C					ofs
CA					Portfolio
US	--		TW	2035	Obama's goal for clean energy

Millions of active endpoints introducing rapid large random fluctuations in supply and demand

DER: PVs, wind turbines, batteries, EVs, DR loads

Control challenges

Need to close the loop

- Real-time feedback control
- Driven by uncertainty of renewables

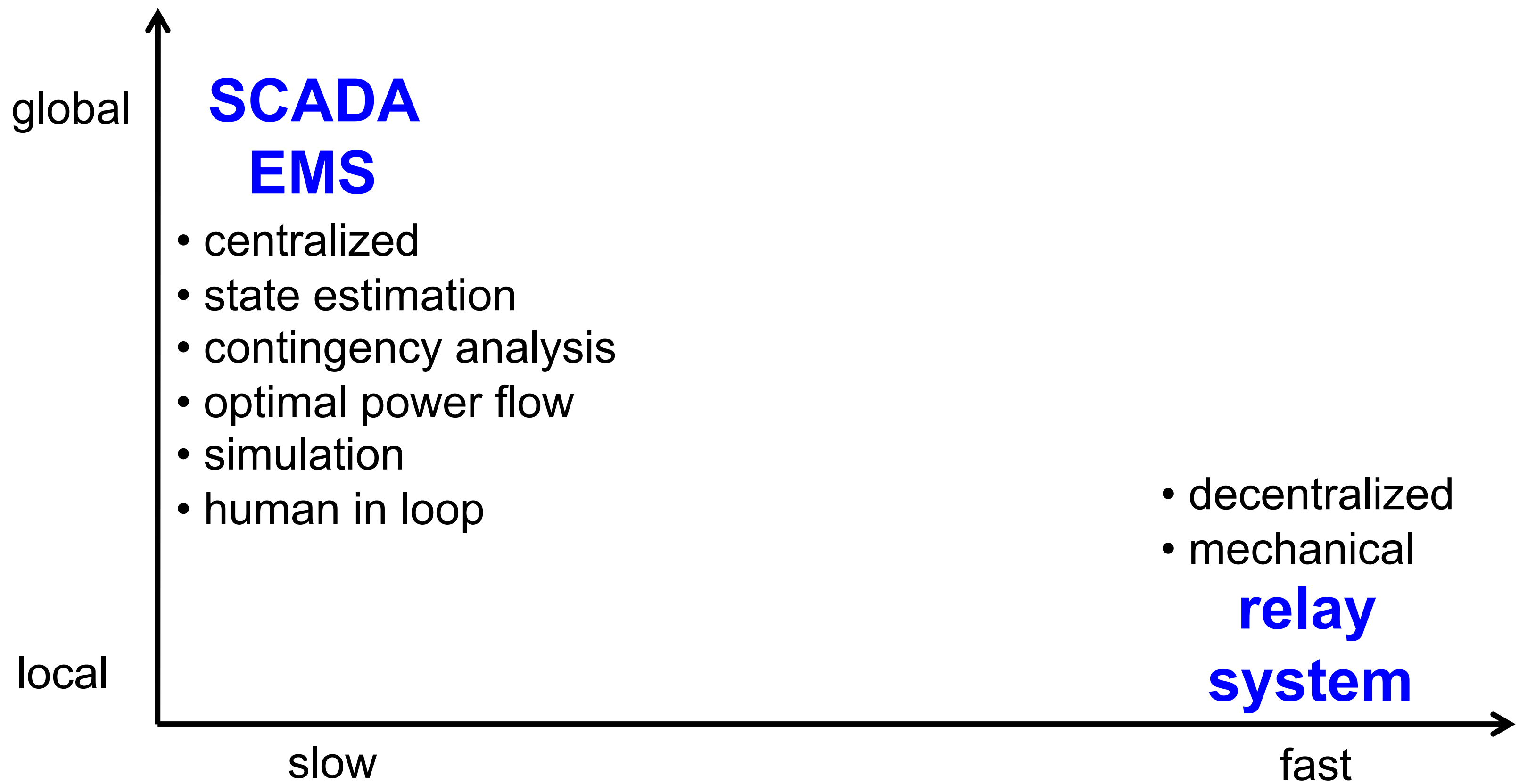
Scalability

- Orders of magnitude more endpoints that can generate, compute, communicate, actuate
- Driven by new power electronics, distributed arch

Engineering + economics

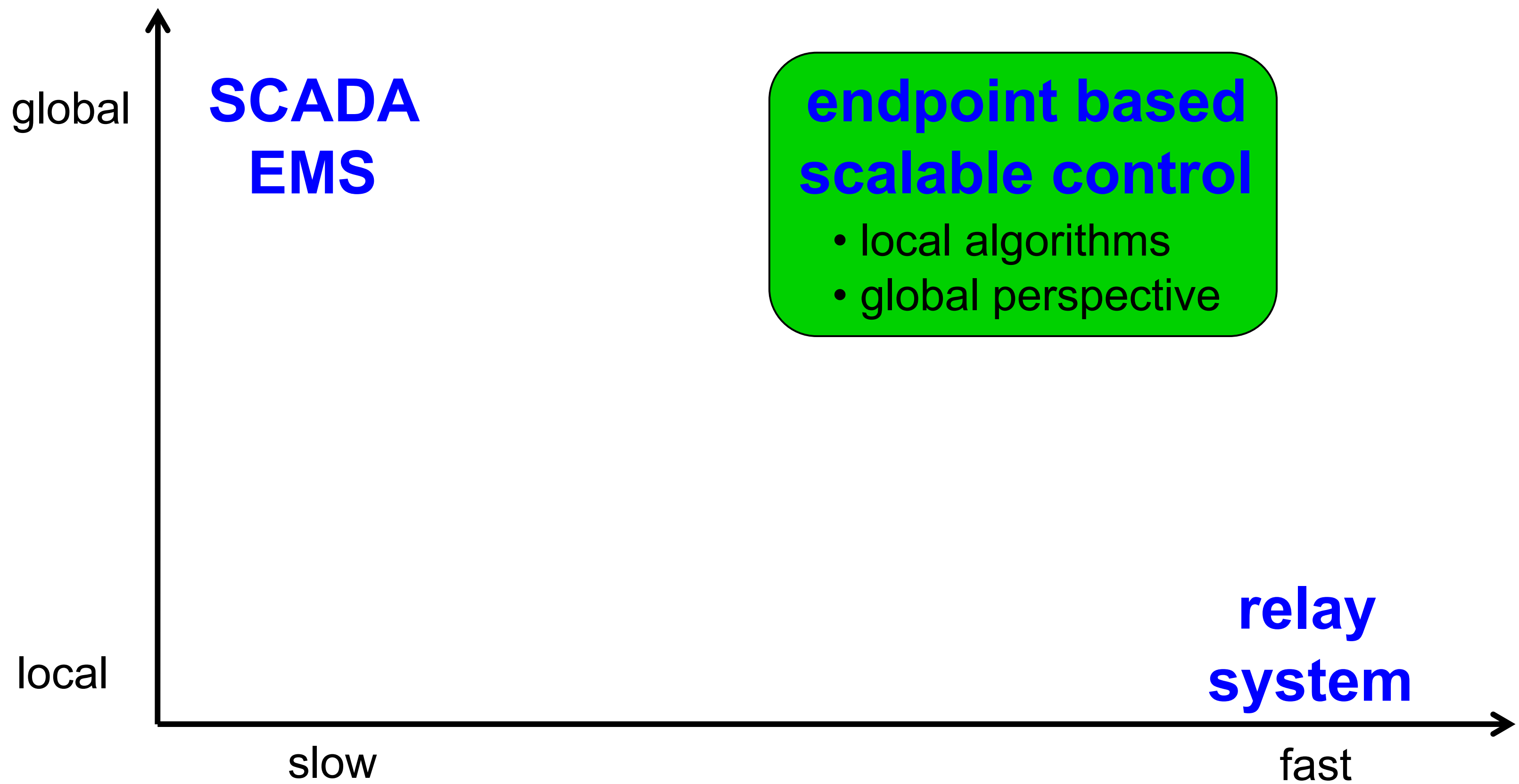
- Need interdisciplinary holistic approach
- Power flow determined by markets as well as physics

Current control



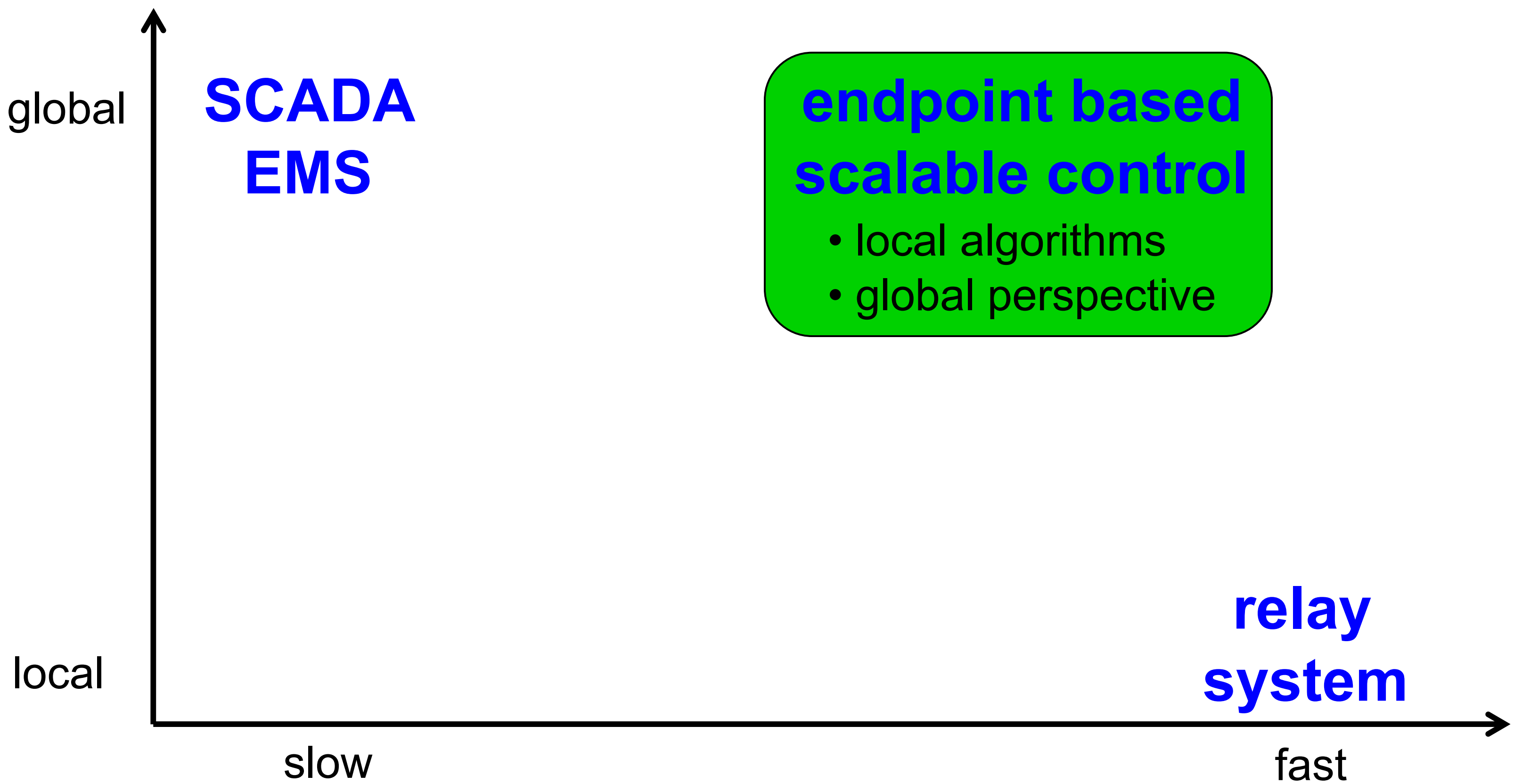
mainly centralized, open-loop preventive, slow timescale

Our approach



scalable, decentralized, real-time feedback, sec-min timescale

Our approach



We have technologies to monitor/control 1,000x faster
not the fundamental theories and algorithms

Our approach

Endpoint based control

- Self-manage through local sensing, communication, control
- Real-time, scalable, closed-loop, distributed, robust

Local algorithms with global perspective

- Simple algorithms
- Globally coordinated

Control and optimization framework

- Systematic algorithm design
- Clarify ideas, explore structures, suggest direction

Ambitious, comprehensive, multidisciplinary
Start with concrete relevant component projects

Our approach: benefits

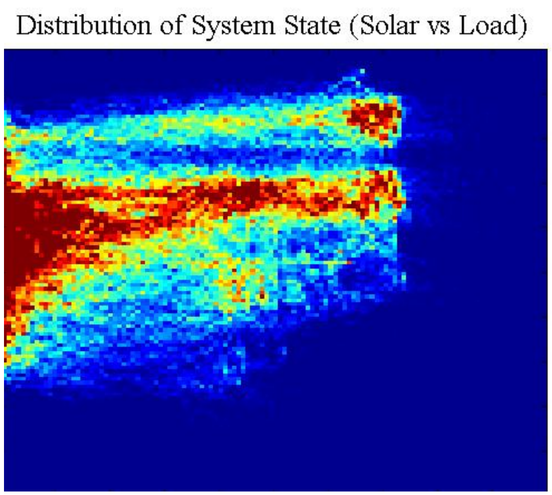
Scalable, adaptive to uncertainty

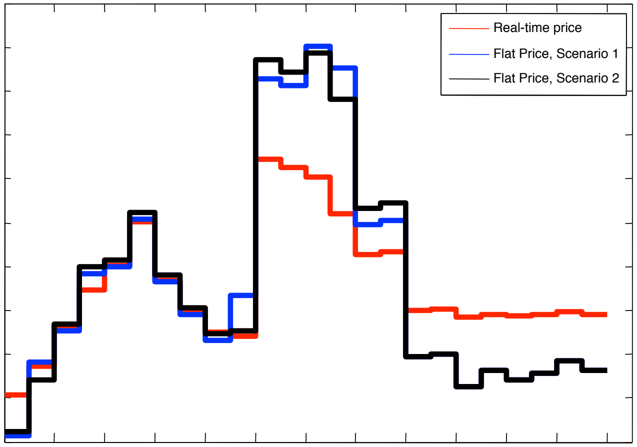
- By design

Robust understandable global behavior

- Global behavior of interacting local algorithms can be cryptic and fragile if not designed thoughtfully

Improved reliability & efficiency





Sample projects

Optimal power flow [Bose, Gayme, L, Chandy]

- **Motivation**: core of grid/market operation, but slow & inefficient computation
- **Result**: zero duality gap for radial networks
- **Impact**: much faster and more efficient algorithm for global optimality to cope with renewable fluctuations

Volt/VAR control [Farivar, L, Clarke, Chandy]

- **Motivation**: static capacitor-based control cannot cope with rapid random fluctuations of renewables
- **Result**: optimal real-time inverter-based feedback control
- **Impact**: more reliable and efficient distribution network at high renewable penetration

Sample projects

Contract for wind [Cai, Aklakha, Chandy]

- **Motivation**: wind producers may withhold generation to maximize profit
- **Result**: simple condition on marginal imbalance penalty incentivizes max wind production
- **Impact**: max renewable power and min market manipulation

Procurement strategy [Nair, Aklakha, Wierman]

- **Motivation**: how to optimally procure uncertain energy
- **Result**: optimal procurement strategy in terms of reserve levels
- **Impact**: Effectiveness of intra-day markets

Sample projects

EV charging [Gan, Topcu, L]

- **Motivation**: uncoordinated charging will produce unacceptable voltage fluctuations and overload
- **Result**: decentralized scheduling that is optimal (valley-filling)
- **Impact**: can accommodate more EV on same grid infrastructure

Frequency-based load control [Zhao, Topcu, L]

- **Motivation**: frequency regulation only by adapting generation can be insufficient
- **Result**: decentralized load control algorithm for supply-demand balancing and frequency regulation
- **Impact**: more responsive frequency regulation in the presence of uncertain supply

Sample projects

Demand response [Na, Chen, L]

- **Motivation**: to maintain power balance
- **Result**: decentralized, scalable, incentive compatible day-ahead scheduling algorithm
- Deterministic case

Stochastic case [Libin Jiang, L]

- Next

Outline

Caltech smart grid research

Optimal demand response

- Model
- Results

Features to capture

Wholesale markets

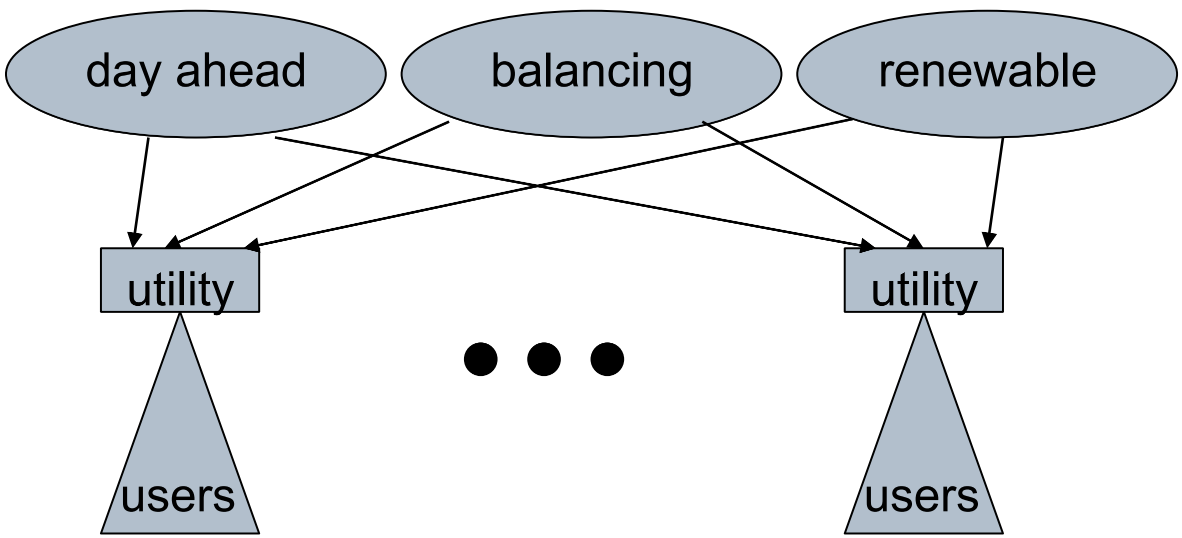
- Day ahead, real-time balancing

Renewable generation

- Non-dispatchable

Demand response

- Real-time control (through pricing)



Model: user

Each user has 1 appliance (wlog)

- Operates appliance with probability $\pi_i(t)$
- Attains utility $u_i(x_i(t))$ when consumes $x_i(t)$

$$\underline{x}_i(t) \le x_i(t) \le \overline{x}_i(t) \qquad \sum_t x_i(t) \ge \bar{X}_i$$

Demand at t:

$$D(t) := \sum_i \delta_i x_i(t) \qquad \delta_i = \begin{cases} 1 & \text{wp } \pi_i(t) \\ 0 & \text{wp } 1 - \pi_i(t) \end{cases}$$

Model: LSE (load serving entity)

Power procurement

- Renewable power: $P_r(t),\ c_r\big(P_r(t)\big) = 0$
  - Random variable, realized in real-time
- Day-ahead power: $P_d(t),\ c_d\big(P_d(t)\big),\ c_o\big(\Delta x(t)\big)$
  - Control, decided a day ahead
- Real-time balancing power: $P_b(t),\ c_b\big(P_b(t)\big)$
  - $P_b(t) = D(t) - P_r(t) - P_d(t)$

capacity
energy

- Use as much renewable as possible
- Optimally provision day-ahead power
- Buy sufficient real-time power to balance demand

Simplifying assumption

- No network constraints

Questions

Day-ahead decision

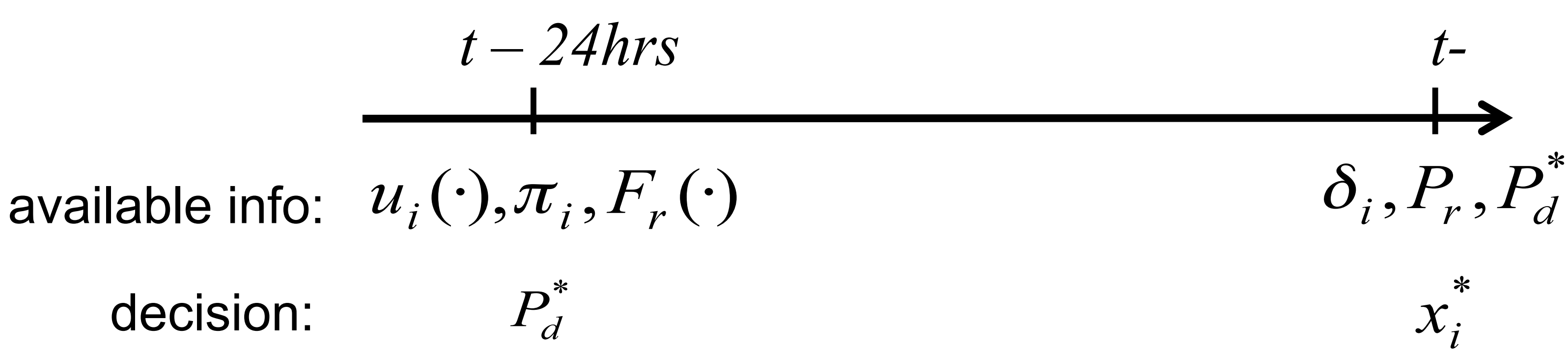
- How much power P_d should LSE buy from day-ahead market?

Real-time decision (at t-)

- How much x_i should users consume, given realization of wind power P_r and δ_i ?

How to compute these decisions distributively?

How does closed-loop system behave ?



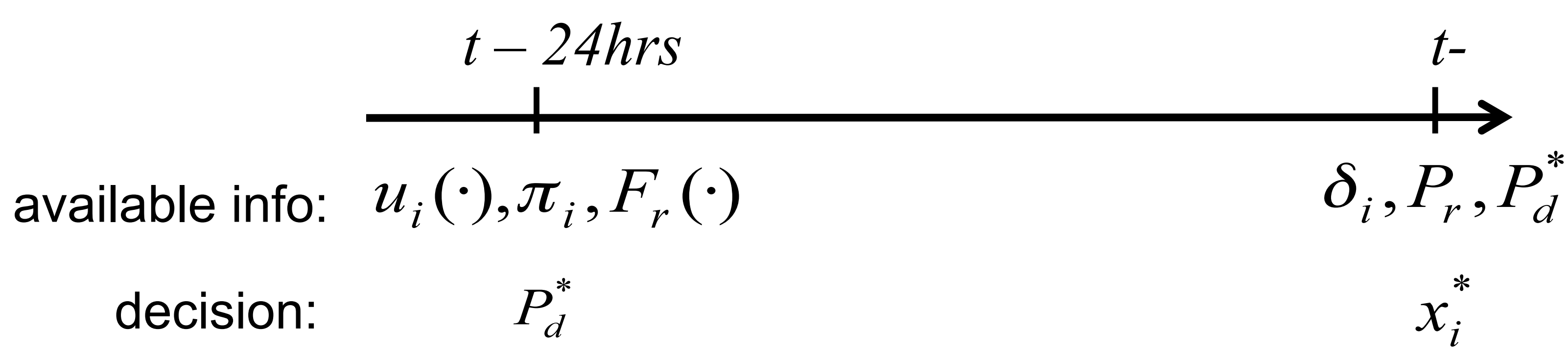
Our approach

Real-time (at *t*-)

- Given P_d and realizations of P_r, δ_i, choose optimal $x_i^* = x_i^*\left(P_d; P_r, \delta_i\right)$ to max social welfare, through DR

Day-ahead

- Choose optimal P_d^* that maximizes expected optimal social welfare



Optimal demand response

Model

Results

- Without time correlation: distributed alg
- With time correlation: distributed alg
- Impact of uncertainty

No time correlation: T=1

Each user has 1 appliance (wlog)

- Operates appliance with probability $\pi_i(t)$
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Demand at t:

$$D(t) := \sum_i \delta_i x_i(t) \qquad \delta_i = \begin{cases} 1 & \text{wp } \pi_i(t) \\ 0 & \text{wp } 1-\pi_i(t) \end{cases}$$

Welfare function

Supply cost

$$c\left(P_d, x\right) = c_d\left(P_d\right) + c_o\left(\Delta(x)\right)_0^{P_d} + c_b\left(\Delta(x) - P_d\right)_+$$

$$\Delta(x) := \sum_i \delta_i x_i - P_r$$ ← excess demand

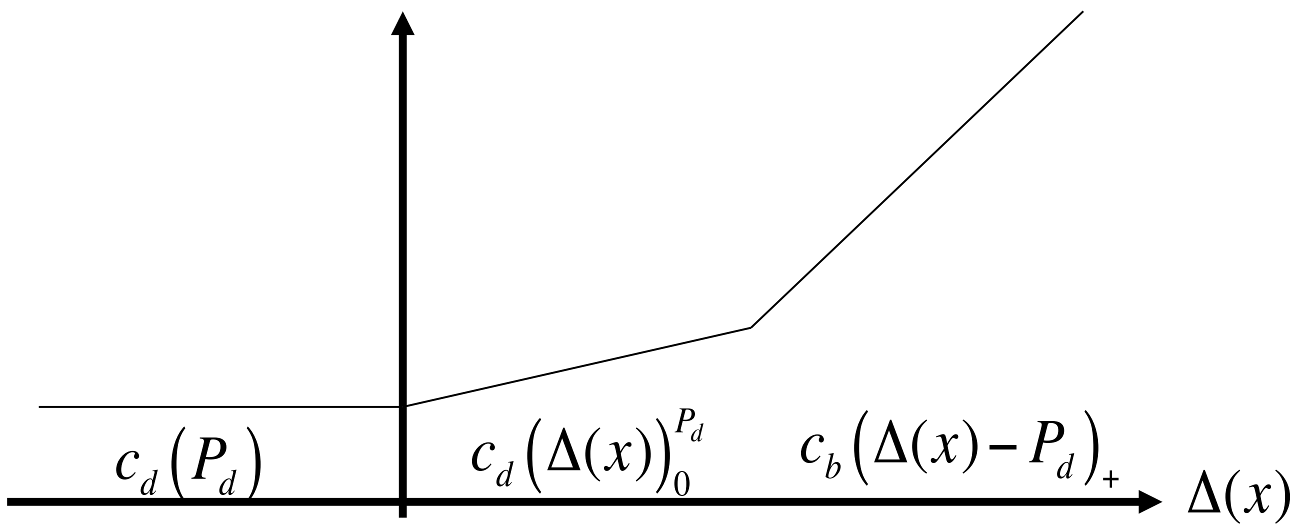
Welfare function

Supply cost

$$c\left(P_d, x\right) = c_d\left(P_d\right) + c_o\left(\Delta(x)\right)_0^{P_d} + c_b\left(\Delta(x) - P_d\right)_+$$

$$\Delta(x) := \sum_i \delta_i x_i - P_r$$

excess demand



Welfare function

Supply cost

$$c\left(P_d, x\right) = c_d\left(P_d\right) + c_o\left(\Delta(x)\right)_0^{P_d} + c_b\left(\Delta(x) - P_d\right)_+$$

$$\Delta(x) := \sum_i \delta_i x_i - P_r$$

← excess demand

Welfare function (random)

$$W\left(P_d, x\right) = \sum_i \delta_i u_i(x_i) - c\left(P_d, x\right)$$

↑ user utility ↑ supply cost

Optimal operation

Welfare function (random)

$$W\left(P_d, x\right) = \sum_i \delta_i u_i(x_i) - c\left(P_d, x\right)$$

Optimal real-time demand response

$$\max_x W\left(P_d, x\right)$$

given realization of P_r, δ_i

Optimal operation

Welfare function (random)

$$W\left(P_d, x\right) = \sum_i \delta_i u_i(x_i) - c\left(P_d, x\right)$$

Optimal real-time demand response

$$x^*\left(P_d\right) \;:=\; \arg \max_x W\left(P_d, x\right)$$

given realization of P_r, δ_i

Optimal operation

Welfare function (random)

$$W\left(P_d, x\right) = \sum_i \delta_i u_i(x_i) - c\left(P_d, x\right)$$

Optimal real-time demand response

$$x^*\left(P_d\right) \quad := \quad \arg \quad \max_x W\left(P_d, x\right)$$

given realization of P_r, δ_i

Optimal day-ahead procurement

$$P_d^* \quad := \quad \arg \quad \max_{P_d} \ \mathrm{E}W\left(P_d, x^*\left(P_d\right)\right)$$

Overall problem: $\max_{P_d} \ \mathrm{E} \ \max_x W\left(P_d, x\right)$

Real-time DR vs scheduling

- Real-time DR: $\max_{P_d} \mathrm{E} \max_x W(P_d, x)$
- Scheduling: $\max_{P_d} \max_x \mathrm{E}\, W(P_d, x)$

Theorem

Under appropriate assumptions:

$$W^*_{real-time\ DR} = W^*_{scheduling} + \frac{N\gamma^2}{1+N\gamma}\sigma^2$$

benefit increases with

- uncertainty σ^2
- marginal real-time cost γ

Algorithm 1 (real-time DR)

$$\max_{P_d} \mathrm{E} \underbrace{\max_x W(P_d, x)}_{\text{real-time DR}}$$

Active user i computes x_i^*

- Optimal consumption

LSE computes

- Real-time "price" μ_b^*

*

Algorithm 1 (real-time DR)

Active user i :

$$x_i^{k+1} = \left(x_i^k + \gamma \left(u_i{}'\left(x_i^k\right) - \mu_b^k \right) \right)_{\underline{x}_i}^{\overline{x}_i}$$

inc if marginal utility > real-time price

LSE :

$$\mu_b^{k+1} = \left(\mu_b^k + \gamma \left(\Delta\left(x^k\right) - y_o^k - y_b^k \right) \right)_+$$

inc if total demand > total supply

- Decentralized
- Iterative computation at t-

Algorithm 1 (real-time DR)

Theorem: Algorithm 1

Socially optimal

- Converges to welfare-maximizing DR $x^* = x^*\left(P_d\right)$
- Real-time price aligns marginal cost of supply with individual marginal utility

$$\mu_b^* = c'\left(P_d, \Delta\left(x^*\right)\right) = u_i'\left(x_i^*\right)$$

Algorithm 1 (real-time DR)

More precisely: $\mu_b^* \in \partial_x c\left(P_d, \Delta\left(x^*\right)\right)$

pricing = marginal cost

$$\mu_b^* \begin{cases} = c_o'\left(\Delta\left(x^*\right)\right) & \text{if } 0 < \Delta\left(x^*\right) < P_d \\ = c_b'\left(\Delta\left(x^*\right) - P_d\right) & \text{if } P_d < \Delta\left(x^*\right) \\ \in \left[c_o'\left(\Delta\left(x^*\right)\right), c_b'\left(\Delta\left(x^*\right) - P_d\right)\right] & \text{if } \Delta\left(x^*\right) = P_d \end{cases}$$

Algorithm 1 (real-time DR)

Theorem: Algorithm 1

Marginal costs, optimal day-ahead and balancing power consumed:

$$c_b'\left(y_b^*\right) = c_o'\left(y_o^*\right) + \mu_o^* \qquad \text{if } P_d^* > 0$$

↑

$$\mu_o^* = \frac{\partial W}{\partial P_d}\left(P_d^*\right)$$

Algorithm 2 (day-ahead procurement)

Optimal day-ahead procurement

$$\max_{P_d} \ \mathrm{E}W\left(P_d, x^*\left(P_d\right)\right)$$

LSE: $$P_d^{m+1} = \left(P_d^m + \gamma^m\left(\mu_o^m - c_d'\left(P_d^m\right)\right)\right)_+$$

calculated from Monte Carlo simulation of Alg 1 (stochastic approximation)

Algorithm 2 (day-ahead procurement)

Optimal day-ahead procurement

$$\max_{P_d} \; \mathrm{E}W\left(P_d, x^*\left(P_d\right)\right)$$

LSE: $$P_d^{m+1} = \left(P_d^m + \gamma^m\left(\mu_o^m - c_d{}'\left(P_d^m\right)\right)\right)_+$$

Given δ^m, P_r^m : $$\mu_o^m = \frac{\partial W}{\partial P_d}\left(P_d^m\right)$$

$$\mu_b^m = \mu_o^m + c_o^{'}\left(y_o^m\right)$$

Algorithm 2 (day-ahead procurement)

Theorem

Algorithm 2 converges a.s. to optimal P_d^*
for appropriate stepsize γ^k

Optimal demand response

Model

Results

- Without time correlation: distributed alg
- With time correlation: distributed alg
- Impact of uncertainty

General T case

Each user has 1 appliance (wlog)

- Operates appliance with probability $\pi_i(t)$
- Attains utility $u_i(x_i(t))$ when consumes $x_i(t)$

$$\underline{x}_i(t) \le x_i(t) \le \overline{x}_i(t) \qquad \sum_t x_i(t) \ge \overline{X}_i$$

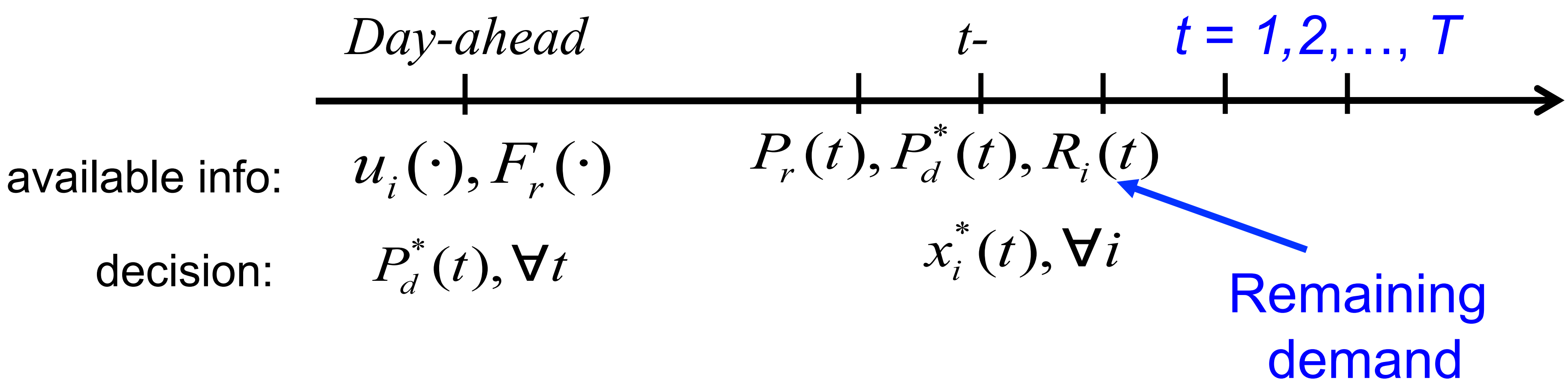
Coupling across time
➔ Need state

Demand at t:

$$D(t) := \sum_i \delta_i x_i(t) \qquad \delta_i = \begin{cases} 1 & \text{wp} \ \ \pi_i(t) \\ 0 & \text{wp} \ \ 1 - \pi_i(t) \end{cases}$$

Time correlation

- Example: EV charging
  - Time-correlating constraint: $\sum_{t=1}^{T} x_i(t) \geq R_i, \forall i$
- Day-ahead decision and real-time decisions



- (1+T)-period dynamic programming

Algorithm 3 (T>1)

- Main idea
  - Solve deterministic problem in each step using conditional expectation of P_r (distributed)
  - Apply decision at current step

- One day ahead, decide P_d^* by solving

$$\max_{P_d, x} \sum_{\tau=t}^{T} W\left(P_d(\tau), x(\tau); \bar{P}_r(\tau)\right) \quad s.t. \sum_{\tau=1}^{T} x_i(\tau) \geq R_i$$

- At time t-, decide $x^*(t)$ by solving

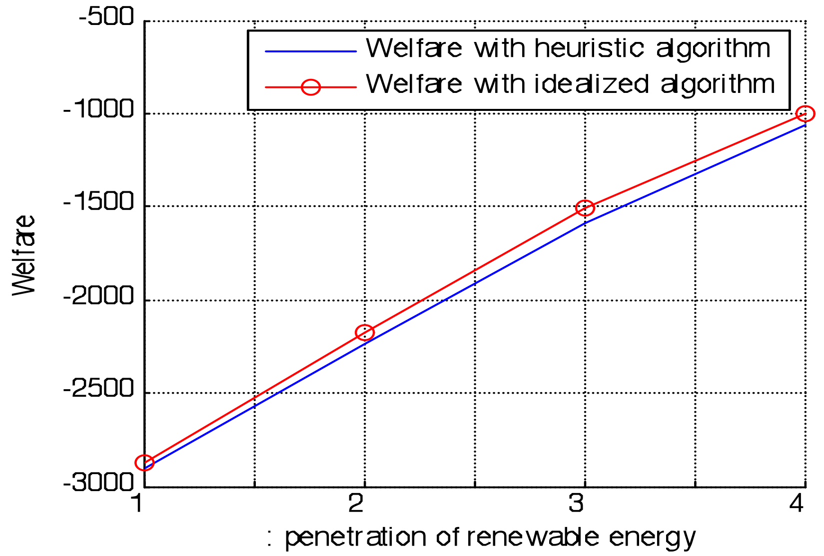
$$\max_{x} \sum_{\tau=t}^{T} W\left(P_d^*(\tau), x(\tau); \bar{P}_r(\tau \mid t)\right) \quad s.t. \sum_{\tau=t}^{T} x_i(\tau) \geq R_i(t)$$

Algorithm 3 (T>1)

<u>Theorem</u>: performance

- Algorithm 3 is optimal in special cases

- $$J^* - J^{A3} \le \sum_{t=1}^{T} \frac{1}{T-t+1} \sigma^2(t)$$



Impact of renewable on welfare

Renewable power:

$$P_r(t;a,b) := a \cdot \mu(t) + b \cdot V(t)$$

mean — zero-mean RV

Optimal welfare of (1+T)-period DP

$$W^*(a,b)$$

Impact of renewable on welfare

$$P_r(t;a,b) := a \cdot \mu(t) + b \cdot V(t)$$

Theorem

- Cost increases in var of P_r
- $W^*(a,b)$ increases in a, decreases in b
- $W^*(s,s)$ increases in s (plant size)